

NUSC Technical Document 7985 8 May 1987



Automated Detection and Tracking Systems for Active Sonar

Roger F. Dwyer Surface Ship Sonar Department



89

LETURN TO DOCUMENTS LIBRARY



Naval Underwater Systems Center Newport, Rhode Island / New London, Connecticut

Approved for public release; distribution is unlimited.

PREFACE

This document was prepared under NUSC Project No. Q25407, "Automated Detection and Tracking Active Sonar," Princial Investigator, R. F. Dwyer (Code 3314); Program Manager, A. Goodman (Code 33B).

The material in this document was presented at a Code 331 technical meeting. Subsequently, the author received many requests for copies of the viewgraphs used in the presentation. This document is meant to satisfy these requests until a more complete report can be finished.

REVIEWED AND APPROVED: 8 May 1987

L. FREEMAN

HEAD: SURFACE SHIP SONAR DEPARTMENT

The author of this document is located at the New London Laboratory, Naval Underwater Systems Center, New London, CT 06320.

SECURITY	CLASSIE	ICATION	OF THIS PAG	: [

REPORT DOCUMENTATION PAGE							
1a. REPORT SECURITY CLASSIFICATION			1b. RESTRICTIVE MARKINGS				
UNCLASSIFIED			Í				
Za. SECURITY CLASSIFICATION AUTHORITY				I/AVAILABILITY O			
2b. DECLASSIFICATION / DOWNGRADING SCHEDULE			Approved for public release;				
28. DECENSION OF WHAT SAME SCHOOLS			distribution is unlimited.				
4. PERFORMING ORGANIZATION F TD 7985	REPORT NUMB	ER(S)	5. MONITORING	ORGANIZATION R	EPORT NUMBER	(\$)	
So MANS OF RESCONDER ORGANIZATION ISL OFFICE CYMPOL			7- 110005 05 06	2011722112		·	
6a. NAME OF PERFORMING ORGANIZATION Naval Underwater		6b. OFFICE SYMBOL (If applicable)					
Systems Center		3314	NUSC 33B (A. Goodman)				
6c. ADDRESS (City, State, and ZIP Code).				ty, State, and ZIP			
New London Laborator							
New London, CT 06320)						
8a. NAME OF FUNDING/SPONSOR	2006	8b. OFFICE SYMBOL	2 22200 12514514	T 10.070, 11.00			
ORGANIZATION	UNG	(If applicable)	9. PROCUREMEN	IT INSTRUMENT ID	ENTIFICATION N	OMBEK	
NUSC		33B					
BC. ADDRESS (City, State, and ZIP C			10. SOURCE OF	FUNDING NUMBER	is		
New London Laborator			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.	
New London, CT 06320)		CEEIMENT NO.	Q25407	100.	Accession no.	
11. TITLE (Include Security Classifi	ication)			<u> </u>	L	_L	
AUTOMATED DETECTION	-	SYSTEMS FOR ACT	IVE SONAR				
12 PERSONAL AUTHOR(S) Roger F. Dwyer						· ····································	
			14. DATE OF REPORT (Year, Month, Day) 15. PAGE COUNT				
Summary FROM		TO	1987 May 8 46				
16. SUPPLEMENTARY NOTATION				١			
17. COSATI CODE			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)				
		19 CLIDICAT TERMS //	Continue on much	a if acceptant and	a interest but but	ot aughes)	
	UB-GROUP	Automated De	tection	Likeli	hood Ratio		
FIELD GROUP S	US-GROUP	Automated De Automated Tr	tection acking	Likeli			
FIELD GROUP S 19. AB\$TRACT (Continue on reven	UB-GROUP se if necessary	Automated De Automated Tra	tection acking number)	Likeli Sequen	hood Ratio tial Detec	tion	
19 ABSTRACT (Continue on reven	SUB-GROUP Se if necessary ificant a	Automated De Automated Tra and identify by block re dvantage to dete	tection acking number) ecting signa	Likeli Sequen als sequenti	hood Ratio tial Detec ally. Gen	tion erally,	
19 ABSTRACT (Continue on reven There is a sign sequential detection	us-GROUP se if necessary ificant a minimizes	Automated De Automated Tra and identify by block of dvantage to det the average de	tection acking number) ecting signa cision time.	Likeli Sequen als sequenti . This is a	hood Ratio tial Detec ally. Gen n importan	tion erally, t require-	
19. ABSTRACT (Continue on reven There is a sign sequential detection ment in tactial sonar	se if necessary ificant a minimizes . However	Automated De Automated Transidentify by block of dvantage to dete the average deep, when sequentic	tection acking number) ecting signa cision time. al analysis	Likeli Sequen als sequenti This is a is coupled	hood Ratio tial Detec ally. Gen n importan with targe	erally, t require- t tracking a	
19 ABITRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se	if necessary ificant a minimizes . However	Automated De Automated Trandidentify by block of dvantage to dete the average detection-track	tection acking number) ecting signa cision time. al analysis ing system i	Likeli Sequenti als sequenti This is a is coupled is obtained	hood Ratio tial Detec ally. Gen n importan with targe which can	erally, t require- t tracking a be applied to	
19. ABSTRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi	ificant a minimizes . However quential-ve system	Automated De Automated Trand identify by block of dvantage to dete the average detection-tracks. The automated	tection acking number) ecting signation time. al analysis ing system in	Likeli Sequenti als sequenti This is a is coupled is obtained as much like	hood Ratio tial Detec ally. Gen n importan with targe which can a human o	erally, t require- t tracking a be applied to perator in	
There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deci	ificant a minimizes. However quential-ve system sion unti	Automated De Automated Trans identify by block of dvantage to detection track of the average detection track of the automated a high level of the automated of	tection acking number) ecting signation time. al analysis ing system ind system act of confidence	Likeli Sequenti als sequenti This is a is coupled is obtained cs much like ce in the ta	hood Ratio tial Detec ally. Gen n importan with targe which can a human o rget is re	erally, t require- t tracking a be applied to perator in ached. On the	
There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deciother hand, target tr	ificant a minimizes. However quential-ve system sion untiacks whice	Automated De Automated Transition Automated Transition Automated Transition Automated the average detection-tracks. The automated a high level of accumulate low	tection acking number) ecting signation time. al analysis ing system ind system act of confidency v levels of	Likeli Sequenti als sequenti This is a is coupled is obtained cs much like te in the ta confidence	hood Ratio tial Detec ally. Gen importan with targe which can a human orget is reare discar	erally, t require- t tracking a be applied to perator in ached. On the	
19 ABITRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deci other hand, target tr A general discus	ificant a minimizes. However quential-ve system sion untiacks whic sion of o	Automated De Automated Transition Automated Transition Automated Transition Automated	tection acking number) ecting signation time. al analysis ing system if d system act of confidence w levels of al detection	Likeli Sequenti als sequenti This is a is coupled is obtained as much like the in the ta confidence n of signals	hood Ratio tial Detec ally. Gen mimportan with targe which can a human orget is reare discarto noise	erally, t require- t tracking a be applied to perator in ached. On the ded. from a	
19 ABSTRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deci other hand, target tr A general discus likelihood ratio form	ificant a minimizes. However quential-ve system sion untiacks whic sion of oulation i	Automated De Automated Trandidentify by block of dvantage to detection tracks. The automated la high level of accumulate low ptimum sequentias given. These	tection acking number) ecting signation time. al analysis ing system id system act of confidence wels of al detection results are	Likeli Sequenti This is a is coupled is obtained is much like is in the ta confidence n of signals e applied to	hood Ratio tial Detec ally. Gen mimportan with targe which can a human orget is reare discarto noise active so	erally, t require- t tracking a be applied to perator in ached. On the ded. from a nar.	
19 ABSTRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deci other hand, target tr A general discus likelihood ratio form Specifically, the per	ificant a minimizes. However quential-ve system sion untiacks whic sion of oulation iformance	Automated De Automated Trandidentify by block of dvantage to detection tracks. The automated la high level of accumulate low ptimum sequentias given. These in terms of false	tection acking number) ecting signation time. al analysis ing system id system act of confidence v levels of al detection results are se alarm pro	Likeli Sequenti als sequenti This is a is coupled is obtained is much like is much like is much like ce in the ta confidence n of signals e applied to	hood Ratio tial Detec ally. Gen mimportan with targe which can a human orget is reare discar to noise active sod false di	erally, t require- t tracking a be applied to perator in ached. On the ded. from a nar. smissal	
19. ABSTRACT (Continue on reven There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deci other hand, target tr A general discus likelihood ratio form	ificant a minimizes. However quential-ve system sion untiacks whic sion of oulation iformance	Automated De Automated Trandidentify by block of dvantage to detection tracks. The automated la high level of accumulate low ptimum sequentias given. These in terms of false	tection acking number) ecting signation time. al analysis ing system id system act of confidence v levels of al detection results are se alarm pro	Likeli Sequenti als sequenti This is a is coupled is obtained is much like is much like is much like ce in the ta confidence n of signals e applied to	hood Ratio tial Detec ally. Gen mimportan with targe which can a human orget is reare discar to noise active sod false di	erally, t require- t tracking a be applied to perator in ached. On the ded. from a nar. smissal	
There is a sign sequential detection ment in tactial sonar powerful automated se both active and passi that it defers a deciother hand, target tr A general discus likelihood ratio form Specifically, the per probability is given	ificant a minimizes. However quential-ve system sion untitacks which sion of oulation iformance for activ	Automated De Automated Trandidentify by block of dvantage to detection tracks. The automated la high level of accumulate low ptimum sequentias given. These in terms of false	tection acking number) ecting signation time. al analysis ing system id system act of confidence v levels of al detection results are se alarm pro- ing with a li	Likeli Sequenti . This is a is coupled is obtained as much like to the ta confidence of signals applied to bability and mited amoun	hood Ratio tial Detec ally. Gen n importan with targe which can a human orget is reare discar to noise active sod false dit of data.	erally, t require- t tracking a be applied to perator in ached. On the ded. from a nar. smissal	
19. ABSTRACT (Continue on revent) There is a sign sequential detection ment in tactial sonar powerful automated seboth active and passithat it defers a deciother hand, target track A general discus likelihood ratio form Specifically, the perprobability is given	if recessary ificant a minimizes . However quential- ve system sion unti acks whic sion of o ulation i formance for activ OF ABSTRACT ZI SAME AS	Automated De Automated Trade Automated Trade Automated Trade Automated Trade Automated	tection acking number) ecting signation time. al analysis ing system id system act of confidence v levels of al detection results are se alarm pro- ing with a li	Likeli Sequenti Inis is a sequenti is coupled is obtained is much like in the taconfidence in of signals applied to bability and mited amoun	hood Ratio tial Detec ally. Gen n importan with targe which can a human orget is reare discar to noise active sod false dit of data.	erally, t require- t tracking a be applied to perator in ached. On the ded. from a nar. smissal	

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

AUTOMATED DETECTION TRACKING SYSTEMS FOR ACTIVE SONAR

INTRODUCTION

The information and viewgraphs in this document were presented at a NHSC/Code 331 technical meeting. A more complete report will be published in the future.

To the first of the control of the c	J
By Detailed	
Arriv.	
Dist .	
A-1	



AUTOMATED DETECTION AND TRACKING SYSTEMS FOR **ACTIVE SONAR**

ROGER F. DWYER

N0211-GA-87(L)-00487.8



Automated detection and tracking systems for both active and passive sonar employ sequential decision analysis to detect and track potential targets. The sequential procedure also drops false tracks from memory.

This is an important operation in order to maintain finite computer memory limits.

The sequential procedure is a natural way to detect a signal. Consider an operator viewing a display. He will announce a target's presence when he's sure a target is there. This is the basic philosophy behind sequential detection. A target is detected only when there's sufficient cause. But unlike an operator, potential tracks due to noise can be discarded by the sequential procedure.

In this document I will present the fundamentals of sequential detection.

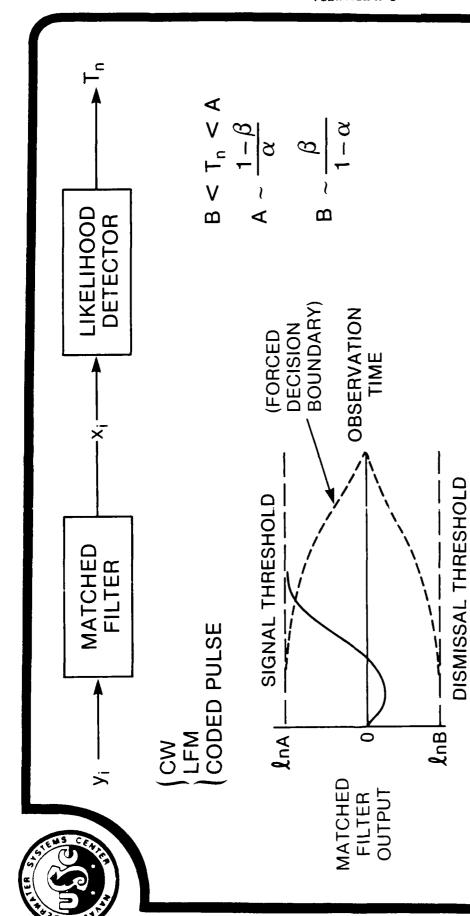
OUTLINE

- INTRODUCTION
- FIXED SAMPLE DETECTION
- SEQUENTIAL DETECTION
- RELATIVE EFFICIENCY
- TRUNCATED TEST (FORCED DECISION)
- GAUSSIAN EXAMPLE
- TWO-DIMENSIONAL GAUSSIAN EXAMPLE
- RAYLEIGH FLUCTUATING TARGET
- CONCLUSIONS

These are the topics that will be discussed in this document. The sequential detector is compared with a fixed sample detector in order to show the sequential detector's optimum property of minimizing the average detection time. The performance measure used is the relative efficiency. A sequential detector with a forced decision boundary is also discussed. These results are important for real tactical situations.

The one-dimensional and two-dimensional Gaussian cases are considered.

Then the Rayleigh fluctuating target model of active sonar is discussed in detail.



- SEQUENTIAL DETECTOR MINIMIZE THE AVERAGE DETECTION TIME
- NEGLECTING EXCESS OVER BOUNDARIES

$$A = \frac{1 - \beta}{\alpha} , B = \frac{\beta}{1 - \alpha}$$

$$\alpha = \frac{1 - B}{A - B} , \beta = \frac{A - 1}{A - B}$$

The sequential detection procedure will be applied at the matched filter output. The active transmission may be CW, LFM, or coded pulses. The likelihood ratio is constructed at the matched filter output to obtain the optimum receiver. Then the sequential analysis procedure is used to detect potential target tracks. Noise tracks are discarded. The advantage of the sequential procedure is that it minimizes the average decision time. This procedure may also be important for long ping durations in surveillance. In this mode a target's presence may be detected before the ping duration is completed giving a tactical advantage to the transmitting ship. However, here I will confine my discussion to multiple transmissions only. This mode is applicable to tactical sonar.

The sequential detection procedure was developed by Abraham Wald. Basically, a decision about the target's presence is deferred until one of two possible boundaries are reached. The loglikelihood ratio output, The, for each ping is compared with two thresholds, lnA and lnB. If In \geq lnA, then a target is present. If In \leq lnB noise only is present and the potential track is dropped from memory. Whereas, if lnB < In < lnA, a decision is deferred until the next ping is received. This procedure minimizes the average detection time. However, the amount of data accumulated is now a random variable. In order to prevent long decision times from occasionally occurring a non-constant decision boundary is sometimes employed. This is called a truncated or forced decision test,

because at some time a decision will be forced to occur. Later I will show that if a truncated test is used the decision time can be reduced further, but at the expense of higher error rates. Nevertheless, this may be a reasonable trade-off. The quantification of a forced decision as given later allows an intelligent trade-off to be made.

Wald was able to relate thresholds in terms of false alarm (α) and false dismissal (β) probabilities by neglecting the excess over the boundaries. This means that the sequential detection procedure terminates on the boundary. This is a good assumption for small signal-to-noise ratio problems. But for high signal-to-noise ratio problems more exact methods are needed.

(Blank page)

Operating Characteristics Function (OCF)

$$L(h) = (A^h - 1) / (A^h - B^h)$$

PROBABILITY L(h) OF ACCEPTING H_0 AS A FUNCTION h.

PROBABILITY OF ACCEPTING H₁

$$= 1 - L(h)$$

h SATISFIES THE CONDITION

$$E[e^{Tnh}] = \int_{-\infty}^{\infty} \cdot \cdot \int_{-\infty}^{\infty} T_n^h f(\underline{x}; s, s,) d\underline{x} =$$

h = h(s, s,); SATISFIED AT $h = \pm 1$

The performance of sequential detection procedures are usually based on the operating characteristic function (OCF) and the average sample number (ASN) as defined by Wald.

The OCF is defined as the probability, L(h), of dismissing the false track as noise (H_0) as a function of the parameter h. Whereas, the probability of accepting the potential track as a target (H_1) is, 1-L(h). This holds because we assume that the test will eventually terminate.

The parameter h itself is a function of the signal. It can be obtained from the characteristic function from the equation $E[e^{Tnh}] = 1$. This equation will always be satisfied at the points, h = 1, h = -1, and h = 0. However, the objective is to find an analytic expression for h which satisfies the above equation for $-1 \le h \le 1$.

Average Sample Number (ASN)

 $\ln R < T_n < \ln A$, CONTINUE DEFERRED DECISION

T_n ≥ InA , TERMINATE, ACCEPT H₁

T_n ≤ lnB , TERMINATE, ACCEPT H₀

ASN =
$$\overline{n} = \frac{L(h) \ln R + [1 - L(h)] \ln A}{E[T_i]}$$

$$\overline{n} = \frac{L(0) (\ln B)^2 + [1 - L(0)] (\ln A)^2}{E[T_i^2]},$$

h = 0

The ASN is defined as the average number of samples (pings) needed to terminate the sequential procedure. The equation for ASN (n) is shown here for $h\neq 0$ and for h=0.

DETECTOR

$$T_N = \sum_i T_i'$$

lim $Pr_q \{ [T_N - E_q (T_N)] / \sigma_q (T_N) \} \rightarrow N (0, 1)$

8 † 8

FIXED SAMPLE

$$N = \{\Phi^{-1} (1-\alpha) - \Phi^{-1} (\beta)\}^{2}/d$$

SEQUENTIAL

$$n = \inf \{n: T_N \not\in (b,a)\}$$

$$E(n|\delta) = \begin{cases} (-2/hd) [b L(\delta) + a (1-L(\delta))], h \neq 0 \\ -ab/d \end{cases}$$

$$h = 1 - 2\delta/\delta_1$$
; $d = [E(T_i'|H_1) - E(T_i'|H_0)]^2 / Var(T_i'|H_0)$

, h = 0

RELATIVE EFFICIENCY

RE = N/E
$$(n|\delta)$$
; h = ±1, h = 0

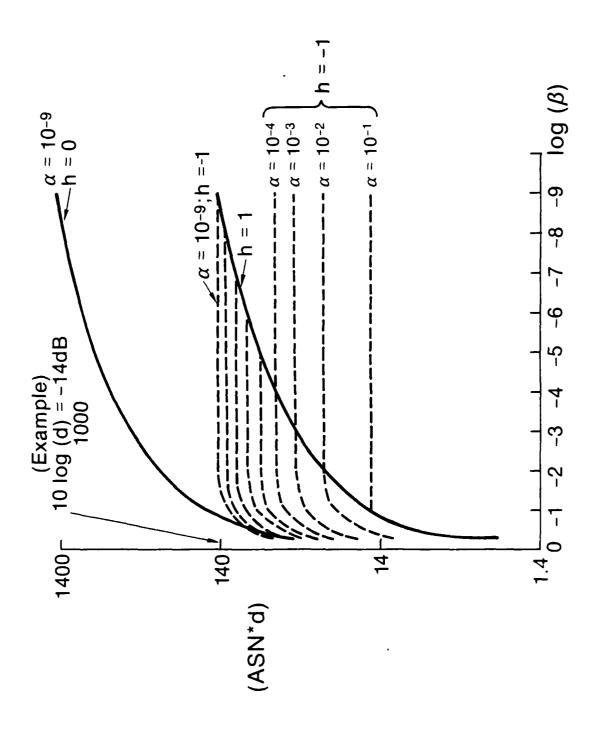
The next 6 figures show asymptotic results. Since it is difficult to obtain general design results for a loglikelihood ratio receiver, because it depends on knowledge of the probability density functions, asymptotic performance results are very useful. Here we assume that the samples (pings) are statistically independent and that the number of samples are sufficiently large to assure that the loglikelihood ratio approaches a Gaussian process. In this way we can compare the performance of a fixed sample detector and a sequential detector using the relative efficiency as a performance measure. However, non-asymptotic results, which will be given later, correspond with these results very well.

The relative efficiency is defined as the ratio

RE = N/E(n)

where, N is the number of samples (pings) required by a fixed sample detector to achieve the desired α and β . Whereas, E(n) is the average number of samples of the sequential detector under the same desired α and β .

Average Sample Number (ASN)



N0211-GA-87(L)-00487-2

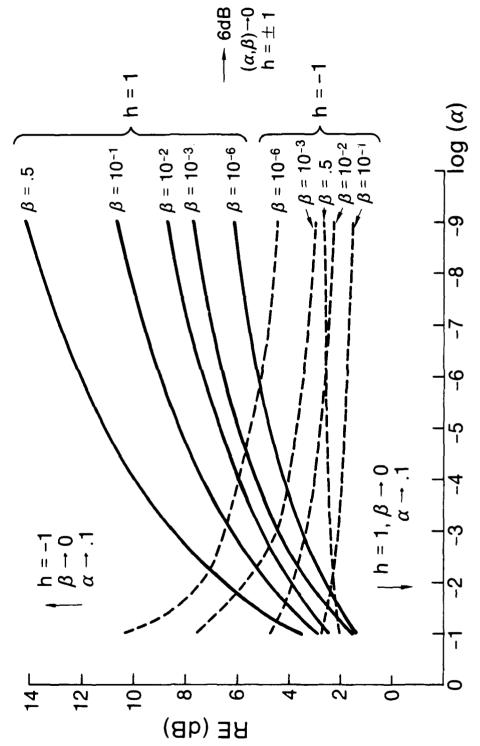
The ASN is defined as the average number of pings needed to make a decision for prescribed α and $\beta.$

The figure gives design curves as a function of α and β . To generalize the results the ASN has been multiplied by d, the signal-to-noise ratio. Three conditions are plotted in the figure, h = \pm 1 and h = 0.

Relative Efficiency at h = ± 1

RE =
$$-(1/2) h (\delta) [\Phi^{-1} (1-\alpha) - \Phi^{-1} (\beta)]^2$$



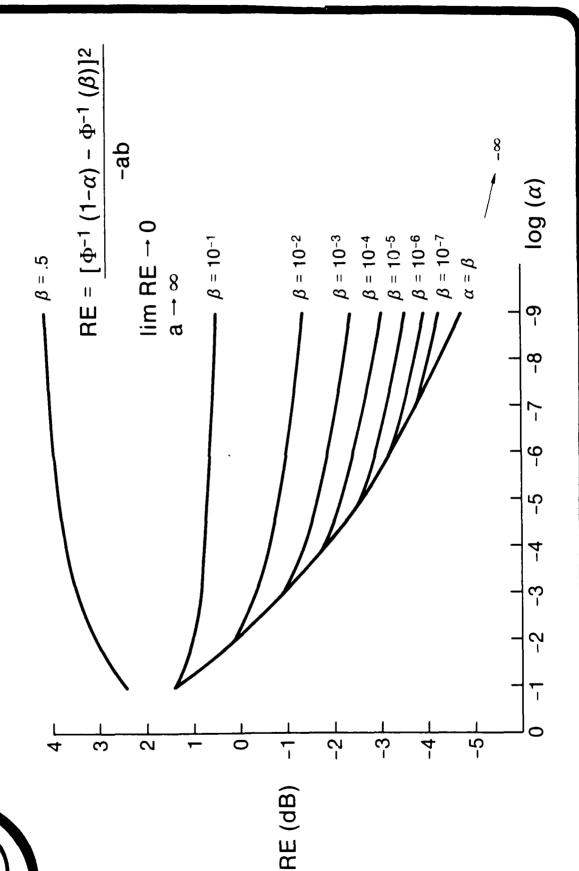


N0211-GA-87(L)-00487 4

In this figure the relative efficiency (RE) is plotted for $h=\pm 1$. As α and β approach zero the RE approached 6 dB. This means that the fixed sample detector requires 4 times as many pings as does the sequential detector to make a decision.

At $\alpha=10^{-8}$ and $\beta=10^{-1}$ then the RE is 10 dB at h = 1, and 2 dB at h = -1. These results are very significant and clearly show the benefit of sequential detection.

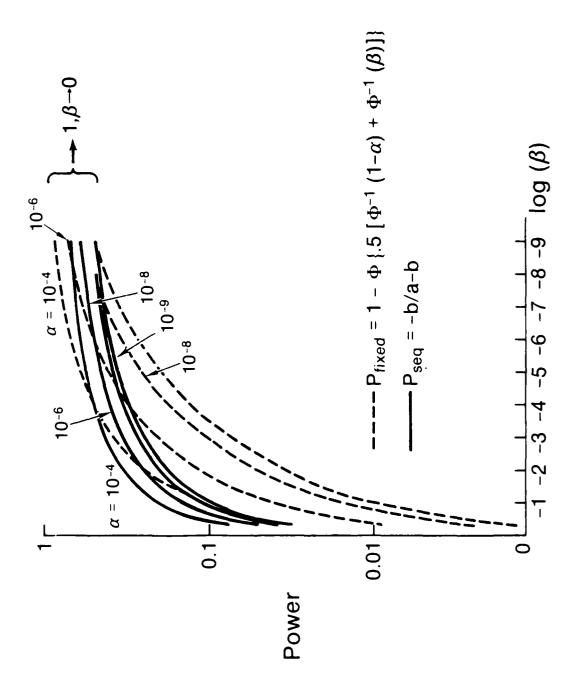




N0211-GA-87(L)-00487 6

This figure shows the RE at h = 0 or the critical point. If α = 10^{-8} and β = 10^{-1} the RE is still greater than one.

Power Comparison at $\delta = \delta_c$



NU211-GA B71L)-00487 5

It is also important to know what the probability of detection (power) is at h=0. This figure compares the power of the sequential detector with the power of the fixed sample detector at h=0.

TRUNCATION

MODIFIED THRESHOLDS

$$b\;G(n) < T_n < a\;G(n)$$
 ,

$$G(n)$$
 = (1 - n/N_T) , o $<$ $G \le 1$, n $<$ N_T

OC FUNCTION

$$L(\delta) = E_a [\exp (aGh) - 1]/\{E_a [\exp (aGh)] - E_b [\exp (bGh)]\}$$

$$L(\delta_c) = a E_a (G)/\{a E_a (G) - b E_b (G)\}\$$

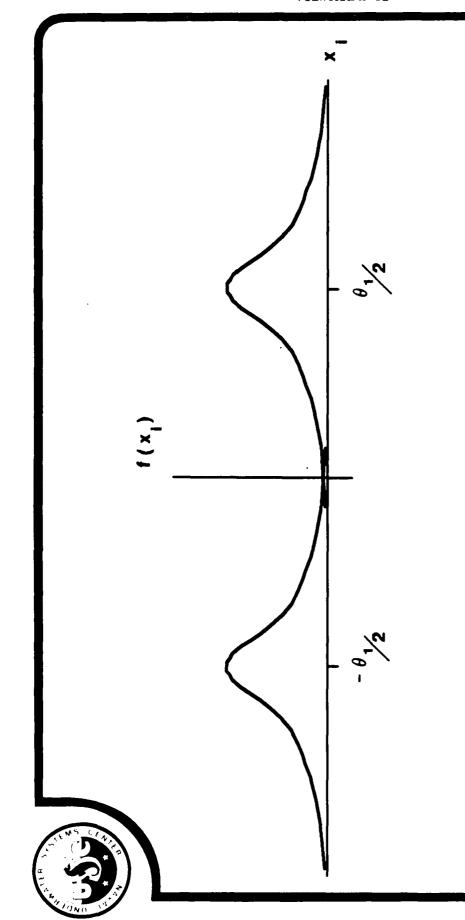
ASN

RESULTS (APPROXIMATE)

0 = 4

$$\mathsf{E}_\mathsf{T}(\mathsf{n}) \geq \mathsf{E}(\mathsf{n})/(1+\mathsf{E}(\mathsf{n})/\mathsf{N}_\mathsf{T}) \leq \mathsf{E}(\mathsf{n}) \left\{ egin{array}{l} \mathsf{h} = -1 \\ lpha_\mathsf{T} \sim lpha \left[(1+\mathsf{a}\,\mathsf{E}(\mathsf{n})/(\mathsf{N}_\mathsf{T}+\mathsf{E}(\mathsf{n})) \right] \end{array}
ight.$$

In a previous figure truncating the sequential detector to force a decision at some point in time was discussed. Here the thresholds are modified by a function G(n). This is a very general approach to the forced decision boundary problem. The performance measures are given in this figure for the modified thresholds. For the special case shown truncation reduces the ASN, but on the other hand the error rates increase. The derived mathematical relationship can now be used to trade-off the number of pings with the increased error rates. These results also hold in general.



, $E[T_i | H_0] = -\theta_1^2/2$; $E[T_i | H_1] = \theta_1^2/2$

 $b < T_n < a$

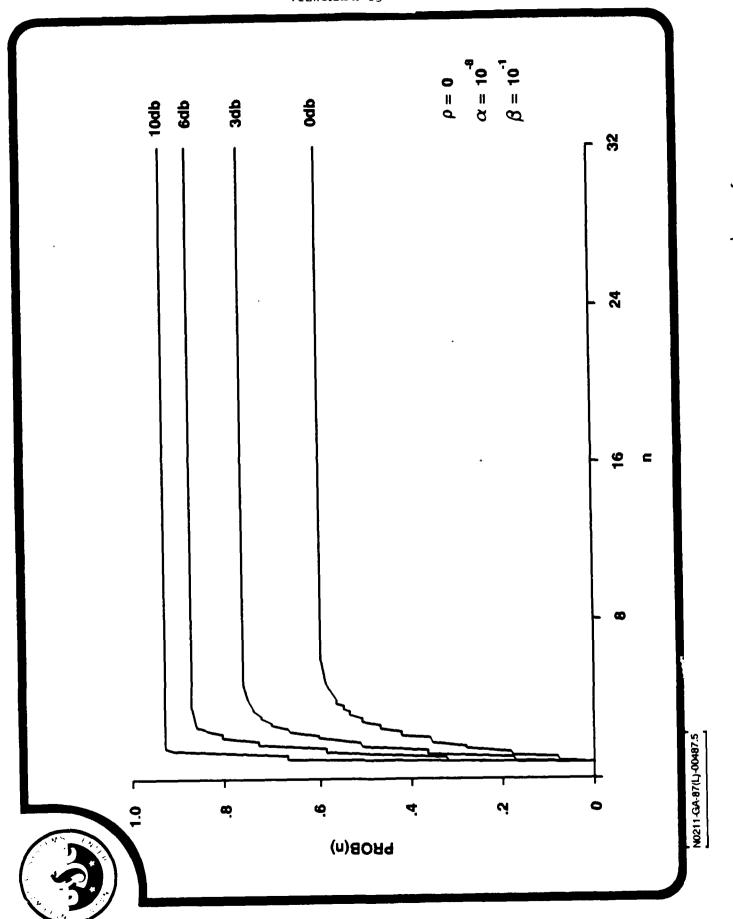
 $T_n = \theta_i \sum_{i=1}^n$

 $T_i = \theta_1 x_i$

N0211-GA-87(L)-00487.7

This figure shows a one-dimensional Gaussian example. Here the matched filter output is a unit variance Gaussian process. The mean value is $\Theta_1/2$ under H_1 and $-\Theta_1/2$ under H_0 . The loglikelihood ratio is employed to obtain T_n .

In the next two figures the results for this example are given. But the data is first quantized into two levels. This is appropriate since information given to operators on displays are quantized and in general computers require quantized data.

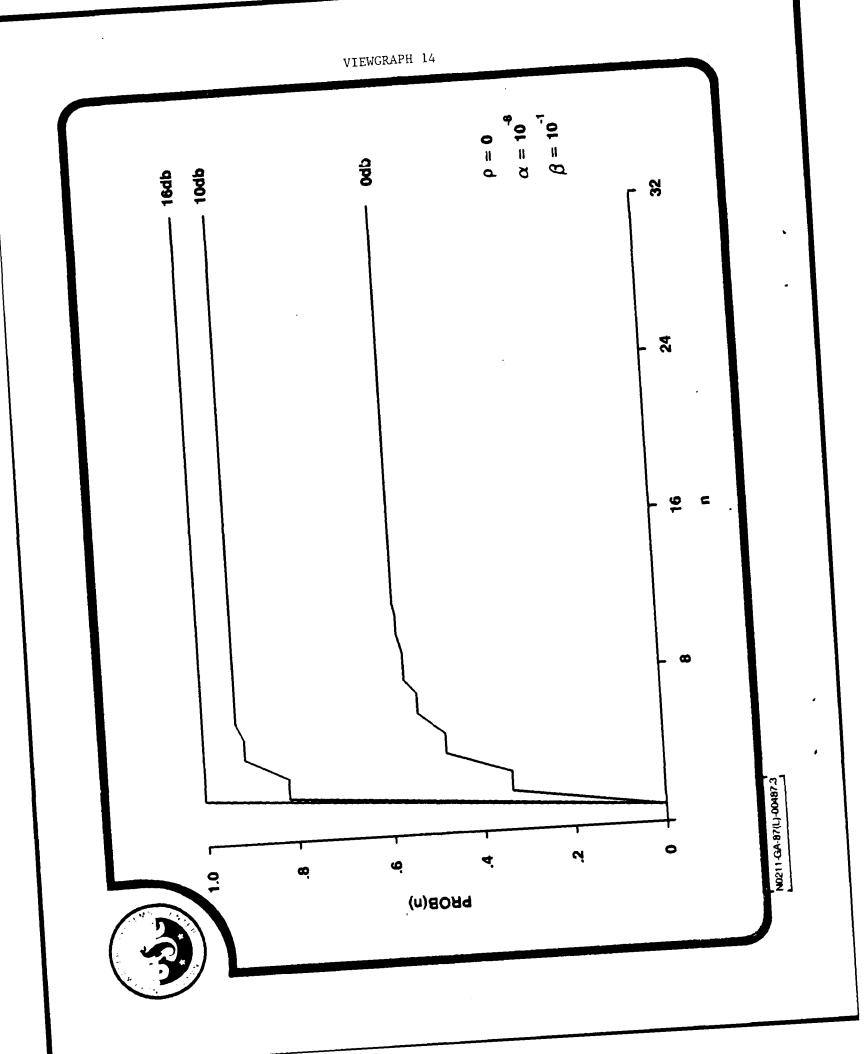


The design parameters for the sequential detector were $\alpha=10^{-8}$ and $\beta=10^{-1}$; whereas the design signal-to-noise (SNR) was 10 dB.

The figure gives the probability of terminating the sequential detector with the acceptance of $H_{\hat{l}}$ as a function of n. The plot is therefore the cumulative distribution function for n.

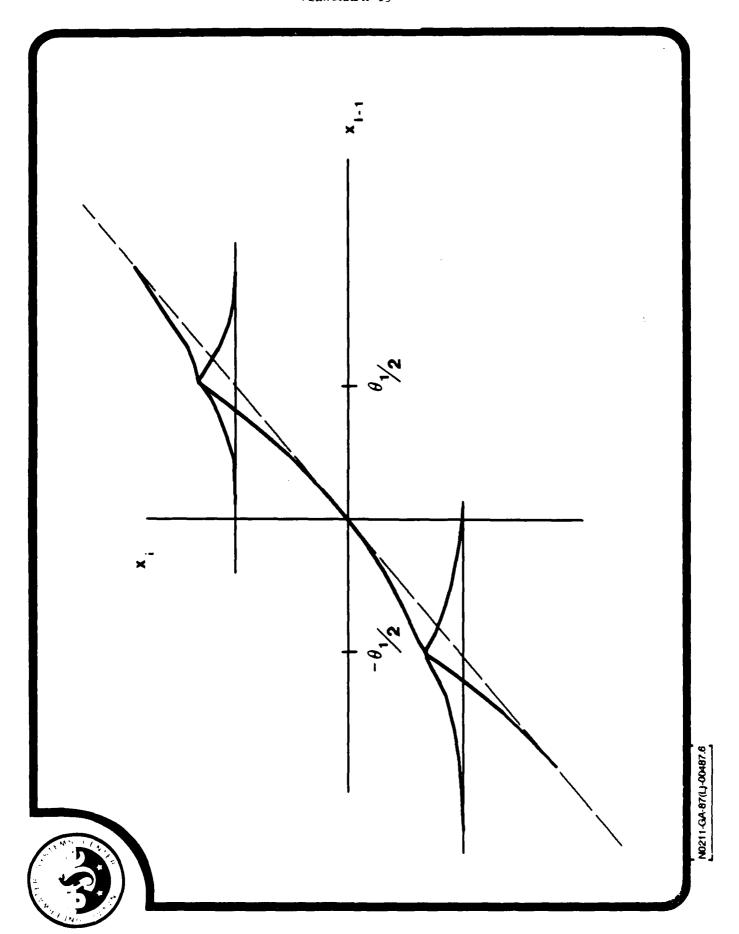
At 10 dB SNR Prob(n) approaches .9 as n increases. Notice that there is a probability of .1 of terminating the test with the acceptance of ${\rm H}_0$, but this is the desired error rate ${\rm B}$.

If the true SNR is less than the design SNR then Prob(n) < .9. This is a general result which holds for all sequential detectors.

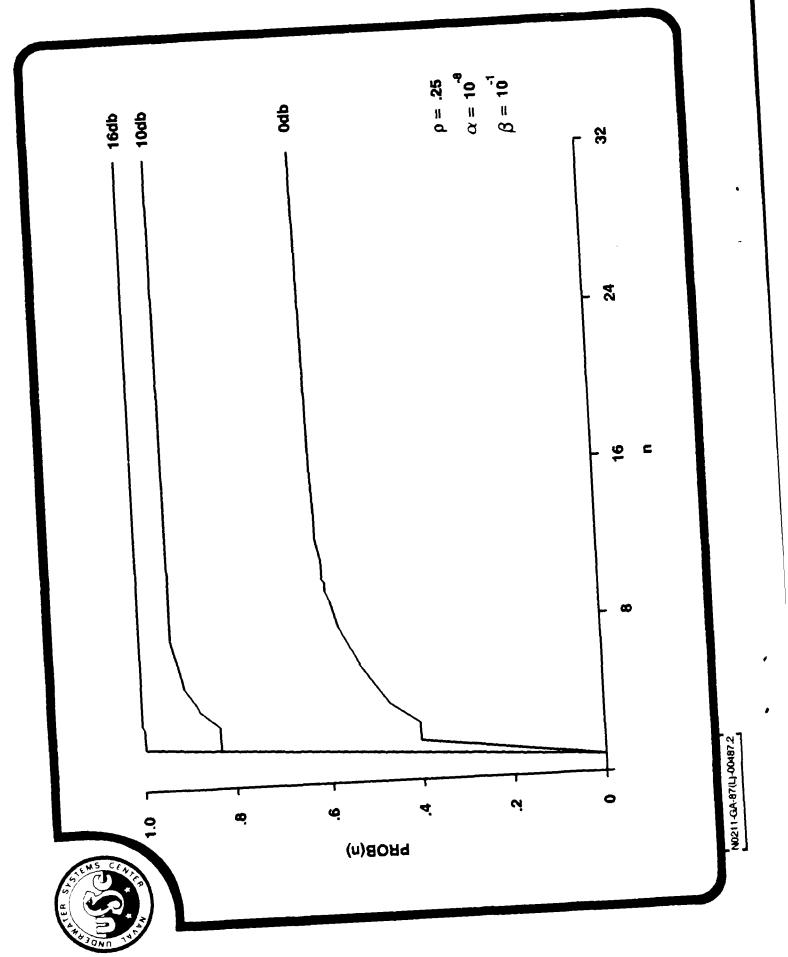


In this figure the design SNR was 16 dB. Now Prob(n) approaches .99 even though α and β are the same as before. The reason this happens is, because, the excess over the boundaries cannot be neglected for high SNR.

As the true SNR decreases from design conditions Prob(n) also decreases.



Now the two-dimensional Gaussian example will be considered. In this case we are assuming that the matched filter output is correlated with the previous ping output. This is called a Markov process. The output of a two level quantizer will be again considered in the following data examples, but now, instead of two regions as in the one-dimensional case, there are 4 quadrants.



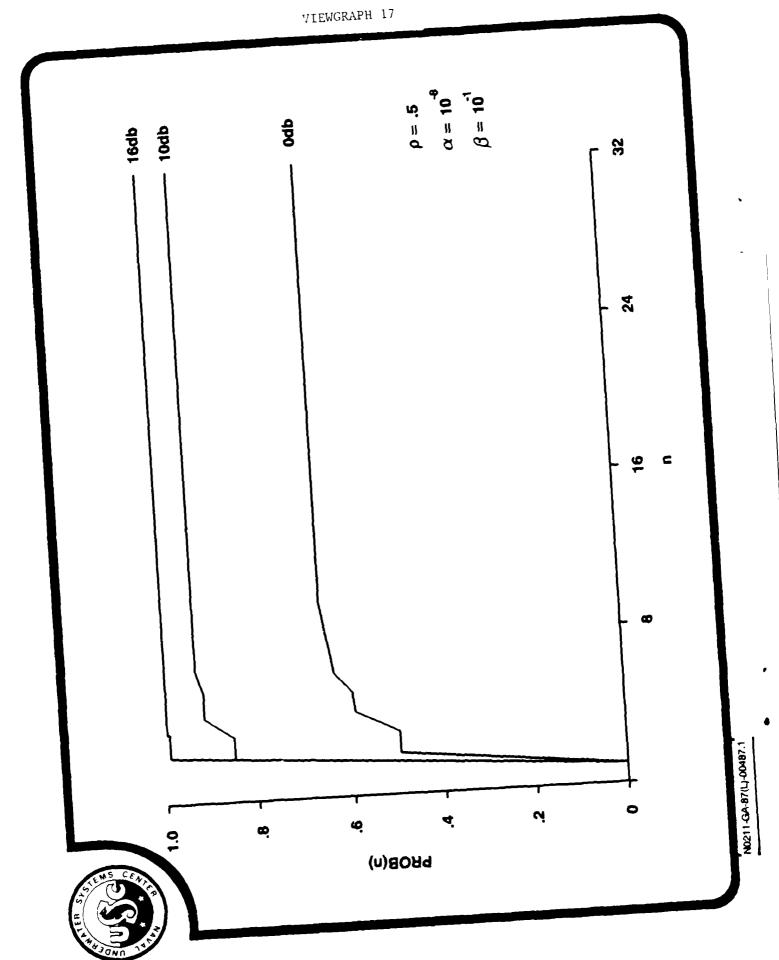
Viewgraph 16, 17, and 18

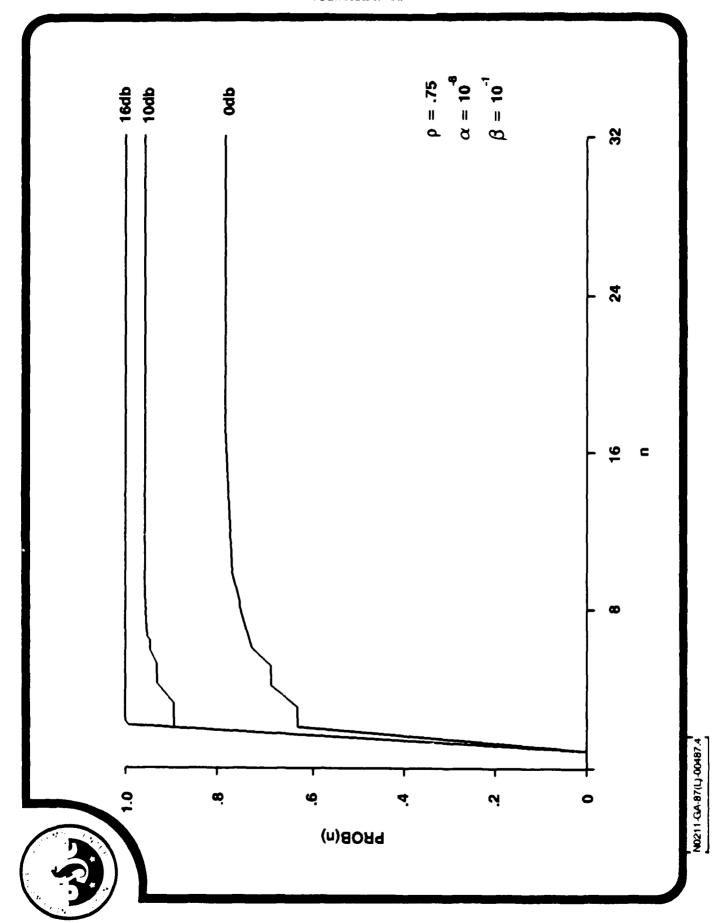
The next three figures show the results for the two-dimensional Gaussian example. Only the correlation coefficients were changed in each figure.

They are .25, .5, and .75, respectively for the next three figures.

The previous figure and the next three figures are for the same high SNR case of 16 dB. However, as the correlation coefficient increases the performance improves for the off design SNR cases. This result may be significant because it represents a method to improve active sonar performance. However, the receiver must be designed from the likelihood ratio in order to achieve improvements. This is true because the likelihood ratio incorporates correlation to achieve an optimum receiver.

As far as the figures are concerned, correlation reduces the false dismissal rates and, thereby, increases detection. Inspection of the likelihood ratio, on the other hand, reveals that correlation shifts its weighting structure to favor detection when correlated data are present. Whereas, potential tracks produced by uncorrelated noise data are quickly rejected.





Rayleigh Fluctuating Target

$$f(x|H_1) = \frac{x}{1+s} \exp(-x^2/2(1+s)) u(x)$$

$$f(x | H_0) = x E(-x^2 / 2) u(x)$$

WHERE, $\sigma^2 = 1 + s$

$$T_i = - \ln(1 + s_1) + \left(\frac{s_1}{1 + s_1}\right) x_i^2 / 2$$
; $E[T_i] = - \ln(1 + s_1) + s_1 \left(\frac{1 + s_1}{1 + s_1}\right)$ (WEAK SIGNAL CASE)

$$E[T_i] = \frac{-s_1^2}{2} + ss_1$$

EXAMPLE:

ASN =
$$\overline{n} = \frac{L(h)b + (1 - L(h))a}{E[T_i]}$$
; $L(1)=1 - \alpha, L(-1) = \beta$

$$s_1 = 16dB$$
, $\overline{n}_0 = .84$, $\overline{n}_1 = .5$
 $s_1 = 10dB$, $\overline{n}_0 = 1.54$, $\overline{n}_1 = 2.1$

$$s_1 = 10dB$$
, $\overline{n}_0 = 1.54$, $\overline{n}_1 = 2.14$
 $s_1 = 6dB$, $\overline{n}_0 = 2.84$, $\overline{n}_1 = 6.8$

$$s_1 = 3dB, \overline{n}_0 = 5.3, \overline{n}_1 = 18$$

The next example is for the Rayleigh fluctuating target. This model is currently employed in active sonar. Therefore, a complete formulation of the sequential detector using Rayleigh statistics are given.

Here the envelope of the matched filter output is represented by x. The probability density functions under H_0 (noise only) and H_1 (signal and noise) are shown. The loglikelihood ratio from these two densities is given by T_1 . This gives the optimum receiver for the Rayleigh fluctuating target model. From this information and the thresholds the ASN can be computed. Several cases are shown. For the high SNR case (S_1 = 16 dB), Wald's formulation does not give an accurate prediction. This is due to the assumption of neglecting the excess over the boundaries. For lower SNR Wald's formulation is very accurate. As SNR decreases more pings are required to detect the target for the prescribed error rates. Notice how the number of pings required falls off as SNR decreases. This is a nonlinear relationship. For only a 3 dB increase in SNR there is a significant decrease in the number of pings required to detect the target.

$$\int_{0}^{\infty} \left[\frac{1}{1+s_{1}} e^{\frac{x^{2}}{2} \left(\frac{S_{1}}{1+s_{1}} \right)} \right]^{h(s, s_{1})} \frac{x}{1+s} e^{-\frac{x^{2}}{2(1+s)}} dx = 1$$

SMALL SIGNAL APPROXIMATION

$$h = 1 - 2s/s_1$$

$$L(h) = (e^{ah} - 1) / (e^{ah} - e^{bh})$$

L(h)	$1-\alpha$.995	o.	.888	.765	$\beta = 1$.
h	-	.25	.01	0	- 1	-

$$a = 18.3$$
, $b = -2.3$

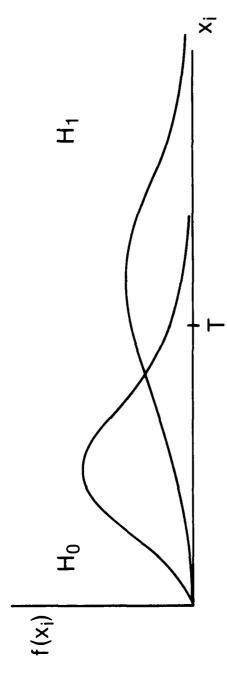
$$\alpha = 10^{-8}$$
$$\beta = 10^{-1}$$

In the previous figure the ASN for the Rayleigh fluctuating target was obtained. Here the operating characteristic function is evaluated for the Rayleigh fluctuating target.

The objective is to solve the integral equation for the parameter h. This integral can be solved. The derived parametric relationship for h, however, is nonlinear with respect with S and S_1 . The small signal approximation is also given. This relationship agrees with the generalized asymptotic results given previously.

Once h is obtained the operating characteristic function can be evaluated given the thresholds. As h varies from 1 to -1, L(h) varies from 1 - α to β .

Some specific results are shown.



$$P_{01} = PROB [x_i < T | H_0]$$

$$P_{02} = PROB[x_i > T | H_0]$$

$$P_{12} = PROB [x_i > T | H_1]$$

 $P_{11} = PROB [x_i < T | H_1]$

$$b_1 = \ln\left(\frac{P_{11}}{P_{01}}\right); \quad b_2 = \ln\left(\frac{P_{12}}{P_{02}}\right)$$

In this example the data at the matched filter is again quantized into two regions. The regions are separated by T in the figure. When the data from a ping falls into one of the regions a value is assigned based on the probability of being in this region under H_0 and H_1 . These values are denoted by b_1 and b_2 in the figure.

The next figure shows the results of the two-level quantizer.

The results of the two-level quantizer example is shown in this figure. The design SNR is 16 dB and α is 10^{-8} and β is 10^{-1} as before. We see that for 16 dB SNR the sequential detector will achieve its design conditions. However, as the SNR decreases from the design SNR the performance falls off and the desired false dismissal rate is not achieved.

CONCLUSIONS

- SEQUENTIAL DETECTORS MINIMIZE AVERAGE **DECISION TIME**
- UTILIZE KNOWLEDGE OF PROBABILITY DENSITY FUNCTIONS TO FORM AN OPTIMUM DETECTOR BASED ON THE LIKELIHOOD RATIO
- SENSITIVE TO PARAMETER AND PROBABILITY DENSITY MISMATCH

Automated detection and tracking systems which utilize sequential analysis are assured of minimizing their average decision time. This is accomplished by incorporating knowledge of the underlying probability density functions based on the likelihood ratio. The resulting automated system acts much like a human operator in that the sequential detector defers a decision until a high level of confidence in the target is obtained. But unlike an operator target tracks which accumulate low levels of confidence are dropped from computer memory. This is an important requirement because it allows additional potential targets to be tracked.

The automated system, however, is sensitive to parameter and probability density function mismatch. There are several methods to overcome these disadvantages. These methods will be discussed in future reports.

References

- 1. A. Nuttall, "Signal Processing in Reverberation A Summary of Performance Capability," NUSC TM No. TC-173-72, 30 August 1972.
- 2. R. Dwyer and L. Kurz, "Characterizing Partition Detectors with Stationary and Quasi-Stationary Markov Dependent Data," IEEE Transactions on Information Theory, Vol. IT-32, No. 4, July 1986.